Exploring Space Through ALGEBRA

EDUCATOR EDITION

Algebra I

Extension Activity: Weightless Wonder

Using TI Graphing Calculators to Evaluate Quadratic Functions
(An Extension Activity to *Weightless Wonder – Reduced Gravity Flight*)

Instructional Objectives

Students will
- select appropriate window settings on a graphing calculator for different functions;
- use a graphing calculator to evaluate quadratic functions; and
- use a graphing calculator to find the maximum value of a quadratic function.

Prerequisites

Students should have prior experience working with the properties of quadratic equations. They should understand the different parts of a parabola and its properties as well as how to find values, such as minimums, maximums, and a vertex point, given a quadratic equation. Students should complete the "Weightless Wonder – Reduced Gravity Flight" activity.

NCTM Principles and Standards

Algebra
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases.
- Draw reasonable conclusions about a situation being modeled.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Grade Level
8-12

Subject Area
Mathematics: Algebra I

Key Concept
Evaluating quadratic functions using the graphing calculator

Teacher Prep Time
5 minutes

Problem Duration
15-30 minutes

Technology
TI-83 Plus family;
TI-84 Plus family; or
other graphing calculator

Materials
- Student Edition
- Graphing Calculator
- *Weightless Wonder – Reduced Gravity Flight* Student Edition

Degree of Difficulty
Moderate

Skill
Use graphing calculators to evaluate quadratic functions; find maximums; recognize *y*-intercepts; connect equation graphs to a real-world problem.

NCTM Principles and Standards
- Algebra
- Problem Solving
- Connections
- Representation

- Monitor and reflect on the process of mathematical problem solving.

Connections
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation
- Recognize and use connections among mathematical ideas.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.

Problem

(Extension from *Weightless Wonder – Reduced Gravity Flight*)

In the Weightless Wonder – Reduced Gravity Flight problem you learned about the parabolic maneuver that NASA pilots perform in order to achieve microgravity during free-fall.

The equation you worked with in *Weightless Wonder – Reduced Gravity Flight* was $y = -4.9t^2 + 87.21t + 9144$. Remember *y* represents the altitude of the plane in meters (m). We will use *x* to represent time in seconds (s), thus making the equation $y = -4.9x^2 + 87.21x + 9144$.

Using a graphing calculator, follow the Guided Instructions below. Then complete the Follow-Up problem.

Lesson Development

This is an extension activity to follow the lesson, Weightless Wonder – Reduced Gravity Flight. Students should refer to their calculations from the problem in that lesson as they are completing this activity.

The purpose of this activity is to familiarize students with many of the capabilities of the graphing calculator. It is divided into two parts: the Guided Instruction and the Follow-Up Problem.

The information given in the guided instruction should be discussed as a class. Students will have varying degrees of expertise using the graphing calculators. Consider having the students model their graphing calculator skills using an overhead projection calculator. This is an ideal situation for students to learn from each other.

The follow-up questions should be done individually.

Guided Instruction

The graphing calculator is a very useful tool to evaluate functions. In *Weightless Wonder – Reduced Gravity Flight,* formulas and paper-pencil calculations were used to evaluate a quadratic function. The following instruction will guide you through the steps needed to find this information using a graphing calculator. It will also verify the answers you derived.

Setting the window to fit a graph

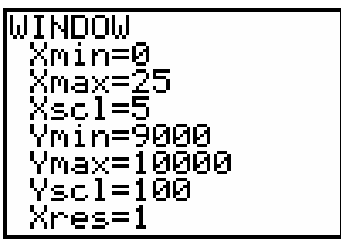
What would be an appropriate window setting for the equation? To answer that question, think about what you know about time mentioned in the background information and in Figure 3 of the *Weightless Wonder – Reduced Gravity Flight* problem.

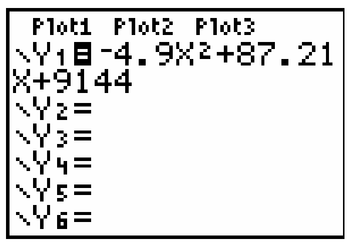
Figure 3 shows the parabola lasting about 25 seconds. So the starting time is 0 seconds and the ending time is at 25 seconds. This tells you the minimum *x* value (Xmin) and the maximum *x* value (Xmax). The scale (Xscl) will place a tick mark on the *x*-axis at certain intervals. Since *x* values are increasing from 0 to 25, 5 might be a good scale.

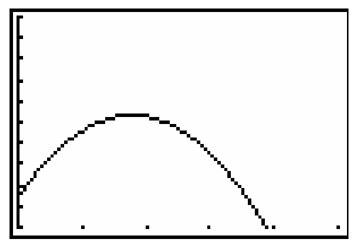
You also know from looking at the equation, $y = -4.9x^2 + 87.21x + 9144$, that the *y*-intercept of the graph is 9,144. Since this is the starting value, you know this is the altitude of the C-9 when it starts the parabolic maneuver. It will increase in altitude before it noses over and begins to dive. Knowing this information, a good minimum *y* value (Ymin) might be 9,000 and a good maximum *y* value (Ymax) might be 10,000. Since the *y* values are increasing from 9,000 to 10,000 a good scale (Yscl) might be 100.

How can you verify the suitability of this window?

Plug in the window settings, place the equation in Y=, and graph it.







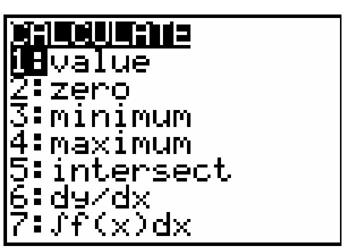
Evaluating functions at a given *x* value.

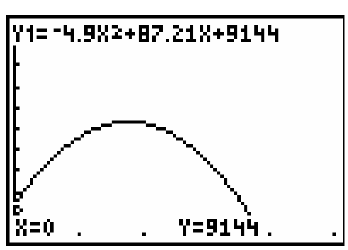
Using the formula given for the parabolic motion, $y = -4.9x^2 + 87.21x + 9144$, how can you use your graphing calculator to verify the altitude where parabolic maneuver began? Can you find the altitude after 3 seconds have elapsed?

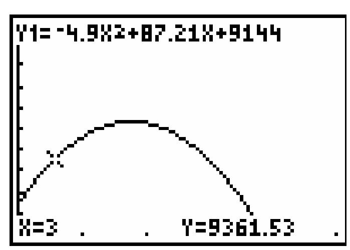
To find the beginning altitude without using the graphing calculator, you would substitute the value of 0 for *x* and solve for *y*. Similarly you could substitute 3 for *x* and solve for *y* to determine the altitude after 3 seconds.

There are a couple of different ways that you can do this with the graphing calculator.

Option 1:
If you use the CALC functions (press 2ND and the TRACE key) you can find the value of *y* at any *x* value by selecting #1: VALUE. When you see the graph on screen, key in the value for *x* and press ENTER.







When $x = 0$ seconds, $y = 9144$ meters and when $x = 3$ seconds, $y = 9361.53$ meters.

Option 2:
Another way of seeing these values is by looking at the table (2ND GRAPH).

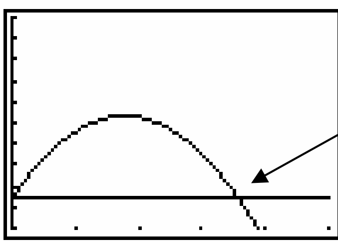
X	Y1	
0	9144	
1	9226.3	
2	9298.8	
3	9361.5	
4	9414.4	
5	9457.6	
6	9490.9	

X=0

The only disadvantage to using the table is that it is harder to find a *y* value when your *x* value is not a whole number.

Finding points of intersection
In Question 2 of *Weightless Wonder – Reduced Gravity Flight*, you were asked to find the length of time the astronaut has in a weightless state given the ending altitude of 9,144 meters. To solve this problem using your calculator, you first need to decide what equations to graph.

The equation $y = -4.9x^2 + 87.21x + 9144$ describes the altitude of the plane where *x* is the time. You want to know what time would give us an altitude of 9,144 meters or when $y = 9144$. So if you graph these two equations you need to find where they intersect, specifically on the way down.

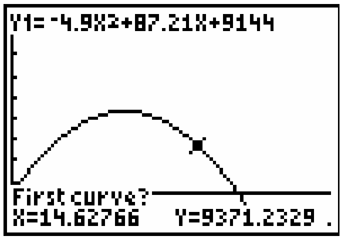


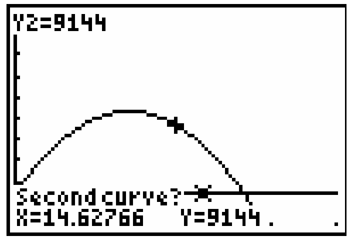
What is the *x* value here?

To find this you can use the CALC functions, found by pressing 2ND and TRACE.
Select #5: INTERSECT and press ENTER.

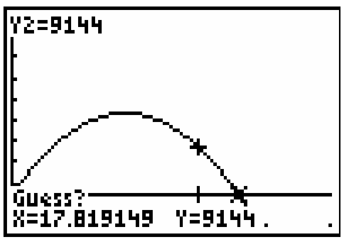
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx

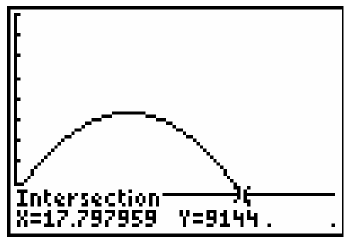
When the screen displays FIRST CURVE, a cursor should be blinking on the parabola. Press ENTER. When it displays SECOND CURVE, it should be blinking on the horizontal line. Press ENTER.





When the screen displays GUESS?, you need to move the cursor to the intersection point that you are trying to find. Notice there are two different places that the two functions intersect. You want the one on the right that represents where the plane is coming down. After moving the cursor, press ENTER and your intersection point will be listed.



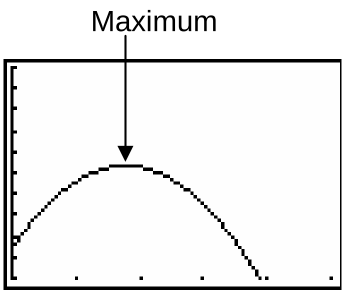


So when *y* (altitude) is 9,144 meters, *x* (time) is 17.798 seconds.

Finding maximum and minimum values

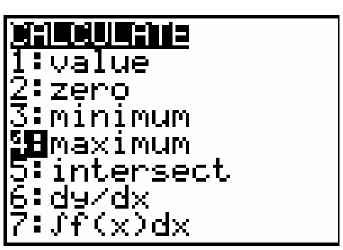
Question 3 of *Weightless Wonder – Reduced Gravity Flight* asks you to find the maximum altitude the plane reaches.

When you look at the graph of the parabola $y = -4.9x^2 + 87.21x + 9144$ on your calculator, you can see approximately where the maximum is.

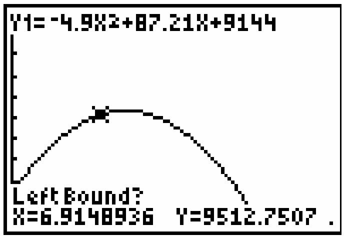


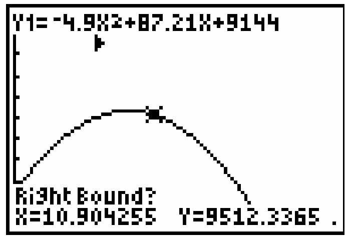
You can use the CALC functions again to find the exact location. The *x* value at this point will tell you the time that the plane reaches the maximum altitude, and the *y* value will tell you the maximum altitude of the plane.

To find a maximum, select #4: MAXIMUM from the CALC menu and press ENTER.

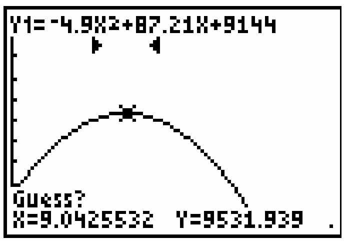


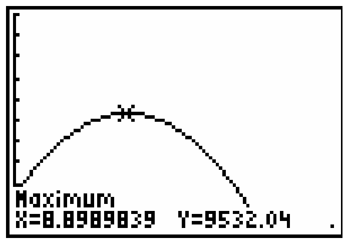
The screen will display LEFT BOUND? This must be a point to the left of your maximum point. Move the cursor so that it is to the left of the maximum and press ENTER. You must now similarly move the cursor to the right of the maximum point when it asks for the RIGHT BOUND?





After you enter the left and right bounds the calculator now asks you to GUESS? where the maximum is. Move your cursor close to that point and press ENTER. The calculator will display the maximum point.





You can see that the maximum altitude is 9532.04 meters and it occurs at 8.899 seconds.

Follow-Up Problem

A C-9 pilot prepares to fly parabolas during a reduced gravity flight. Due to unfavorable weather conditions, the pilot starts the parabolic maneuvers at a lower altitude of 9,000 meters.

Students should use their graphing calculators to answer each question, applying the skills they just learned in the previous guided instruction.

Wrap-Up

Once students complete the entire activity, have them form pairs to discuss the procedures used in the follow-up question. This allows students the opportunity to communicate the skills learned.

Solution Key (One approach)

1. In the previously described reduced gravity flight (see Guided Instruction), the equation of the parabolic motion was $y = -4.9x^2 + 87.21x + 9144$. Assuming the altitude is the only factor that will change, write an equation to describe the parabolic motion of this reduced gravity flight.

   The starting value or *y*-intercept is the only change, thus the new equation is $y = -4.9x^2 + 87.21x + 9000$.

2. The time that elapses during the parabolic motion is still 17.798 seconds. Find the altitude during this time.

   Use the value option of the CALC menu or the table to find the altitude at 17.798 seconds is 9,000 meters.

3. Find the maximum altitude of the plane and the time at which this occurs.

   Use the maximum option from the CALC menu to find the maximum altitude is 9388.04 meters and occurs at 8.899 seconds.

4. Find the occurrences when the plane is at an altitude of 9,300 meters.

   Graph $y = -4.9x^2 + 87.21x + 9000$ and y = 9,300 in the calculator, then use the intersect option from the CALC menu to find the two times are 4.660 seconds and 13.138 seconds.

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Thanks to the subject matter experts for their contributions in developing this problem:

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